Weight Optimization Methods in Space Radiation Shield Design

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An empirical relation between proton range and material density is used to examine relations between shield weight, geometry, and material composition for shielding against a space proton environment. The optimum material resulting in minimum shield weight usually lies at the extremes of either the lightest or heaviest materials. Aluminum, which has special prominence in the space program, appears universally suboptimal as a radiation shielding material. Assuming square-box geometry (rectangular prisms with two square faces), the optimum shape for the shielded object is found to be a cube, although moderate deviations from a cube result in only a small weight penalty.

Introduction

THE short-term nature of the exploratory missions which have characterized most of the United States space program has traditionally simplified the radiation protection problem to evaluation of inherent vehicular radiation protection, judicious trajectory planning, and contingency plans in case of a rather unlikely solar flare event.[1,2] This mode of radiation protection planning was followed, since on-board equipment arrangement and the required wall structure for micrometeoroid protection provided sufficient radiation shielding to meet the mission dose constraints.[3] With the advent of long-duration space flight, the radiation protection problem takes on greater significance, since the inherent vehicular shield effectiveness is no longer adequate to provide sufficient radiation protection.[4-6] As in Skylab, large amounts of additional shielding material will be required in many future missions in which long-duration flight and routine operation in the earth's radiation belts will be typical.

The materials for shielding in the space program have traditionally been chosen for reasons such as meteoroid protection, structural integrity, and environmental control.[1,2] Although some pioneering efforts were made toward shield optimization through choice of material composition,[7,8] such methods were not fully developed, since they would play no essential role during the initial phase of the space program. In distinction, optimization methods based on mass distribution have played an essential role in increasing the inherent vehicular radiation protection,[1] and generalized optimization programs of this type have been developed.[7] The recent Skylab mission and future space prospects have greatly altered shield design considerations and clearly indicate the need and desirability of minimum weight optimization techniques of space radiation shield design.

Simplified Optimization Model

We consider here the simplest form of the optimization problem. Namely, we assume that the shielded region is a square box of arbitrary length (rectangular prism with two square faces) and the thickness of a required reference shielding material is known to maintain exposure to acceptable levels. The required thickness of the reference shielding material corresponds to some proton cutoff energy. Our present task will be to determine which material shield with thickness of the corresponding proton cutoff energy results in the smallest total shield mass. The optimum shape (i.e., long and slender or short and flat) within the square-box geometry for a fixed volume of shielded region will also be considered.

Physical Data

To formulate the problem properly, we require some relation between shield thickness and material composition. The range $R_p(I, \rho_e, E)$ of a proton of energy E is approximately a function of the average ionization potential I and electron density ρ_e.[10] For the shielding materials in Table 1, the relation of proton range is further simplified in that it may be related to energy E and material density ρ to a good approximation

$$R_p(I, \rho_e, E) \approx R_a(\rho, E) \tag{1}$$

where $R_a(\rho, E)$ is the appropriate approximate function. Fur-

Table 1 Material properties used in present analysis in comparison to empirical density-range relation as shown in Fig. 1

Material	R_a (ρ, 100 MeV) (g/cm^2)	ρ, density (g/cm^3)	$R_a(\rho,E)/R(E)$	$F(\rho)$
Polyethylene	7.2	0.92	0.60	0.56
Water	7.6	1.00	0.63	0.58
Polystyrene	7.7	1.06	0.64	0.59
Nylon	7.6	1.13	0.63	0.62
Stilbene	8.1	1.16	0.68	0.63
Saran	9.3	1.69	0.78	0.72
Magnesium	9.6	1.74	0.80	0.73
Beryllium	9.3	1.80	0.78	0.74
Bone	8.2	1.85	0.68	0.75
Carbon	8.5	2.20	0.71	0.79
Silicon	9.67	2.33	0.81	0.80
Aluminum	9.9	2.70	0.83	0.83
Titanium	11.0	4.54	0.92	0.91
Vanadium	11.3	6.00	0.94	0.94
Zinc	11.7	7.13	0.98	0.96
Chromium	11.1	7.19	0.93	0.96
Manganese	11.3	7.43	0.94	0.96
Iron	11.2	7.87	0.93	0.97
Stainless steel	11.2	7.93	0.93	0.97
Nickel	11.2	8.90	0.93	0.97
Copper	11.8	8.92	0.98	0.97

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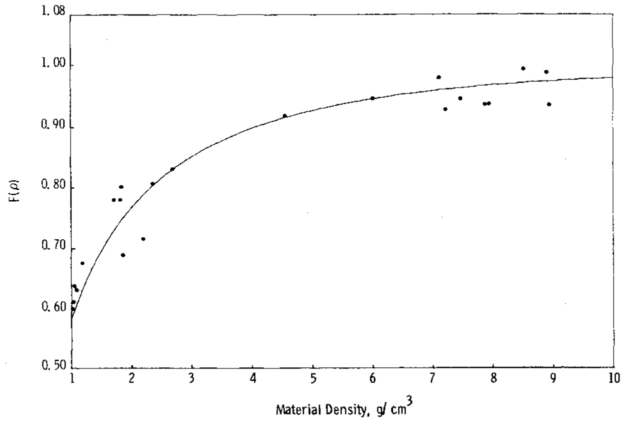

Fig. 1 Normalized proton range for fixed energy as a function of material density.

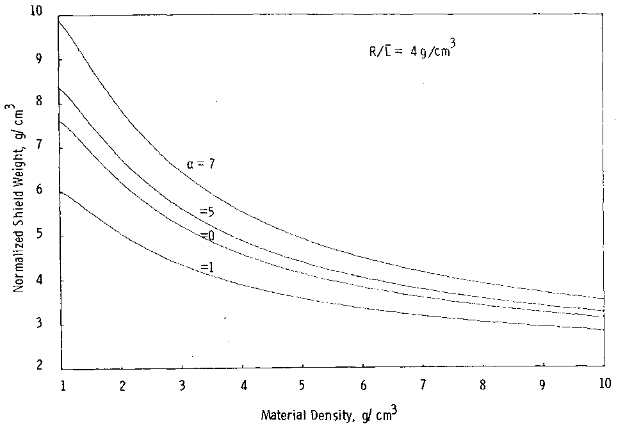

Fig. 2 Normalized shield weight for moderately small shielded regions as a function of shape and material density.

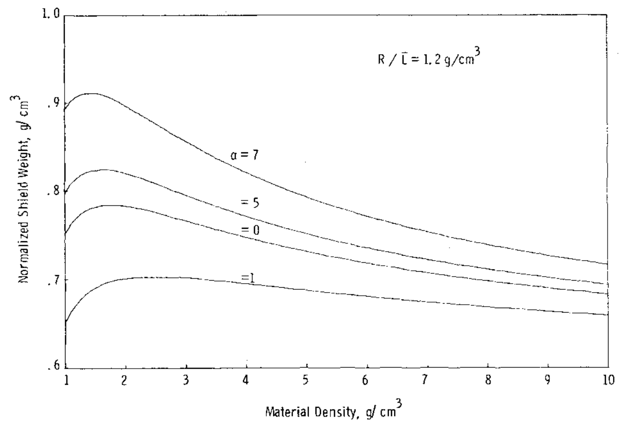

Fig. 3 Normalized shield weight for shielded region dimensions on the order of shield thickness as a function of shape and material density.

thermore, using the equivalent-thickness approximation, we may write

$$R_a(\rho,E) \approx F(\rho)R(E) \tag{2}$$

where an empirical expression for the equivalence function is

$$F(\rho) = 1 - \frac{5}{3} \exp\left(-1.386\sqrt{\rho}\right) \tag{3}$$

From Hill et al.[11] we find the universal range function to be

$$R(E) = 556 \ln\left(1 + 5.48 \times 10^{-6} E^{1.8}\right) \tag{4}$$

where E is in MeV and $R(E)$ is in units of areal density (g/cm^2). The equivalent thickness approximation is discussed elsewhere[12,13] and the empirical fit for $F(\rho)$ is shown in Fig. 1 in comparison with values calculated from Table 1, which are based on the work of Janni[10] for proton range at 100 MeV.

Shield Weight

We consider a shielded region in the shape of a square box of arbirary length denoted by the dimensions

$$L_x = L_y = L_z/\alpha \equiv L \tag{5}$$

where α lies between zero for a flat plate and infinity for a very long box. We define the average length as

$$\bar{L} = \tfrac{1}{3}(2+\alpha)L \tag{6}$$

and the area is given by

$$A = 18\ (1+2\alpha)\bar{L}^2/(2+\alpha)^2 \tag{7}$$

The shield volume is, then

$$V_s = tA + 12\ t^2\bar{L} + 8\ t^3 \tag{8}$$

where the thickness t depends on the proton cutoff energy E and material composition, which is characterized by density ρ, so that

$$t = F(\rho)R(E)/\rho \tag{9}$$

The total shield weight is then

$$W_s = F(\rho)R(E)A + 12\frac{F^2(\rho)}{\rho}R^2(E)\bar{L} + 8\frac{F^3(\rho)}{\rho^2}R^3(E) \tag{10}$$

It will prove convenient to rewrite Eq. (10) for the normalized shield weight as the following

$$\frac{W_s}{\bar{L}A} = F(\rho)\frac{R(E)}{\bar{L}} + \tfrac{2}{3}\frac{F^2(\rho)}{\rho}\frac{(2+\alpha)^2}{(1+2\alpha)}\left[\frac{R(E)}{\bar{L}}\right]^2 + \frac{4}{9}\frac{F^3(\rho)}{\rho^2}\frac{(2+\alpha)^2}{(1+2\alpha)}\left[\frac{R(E)}{\bar{L}}\right]^3 \tag{11}$$

We see that, for a given geometry (i.e., α and $\bar{L}$) and a given cutoff energy, the shield weight is an explicit function of material density. To find the ρ which corresponds to the smallest weight then constitutes the optimal shield material.

Optimality

Examination of the normalized weight equation—Eq. (11)—reveals that for a given ρ only α and $R(E)/\bar{L}$ enter as parameters. The dependence of shield weight on the material density is shown for fixed values of $R(E)/\bar{L}$ and α in Figs. 2-5. Figure 2 corresponds to shielding of an object which is relatively small compared to the shield size (i.e., the object's dimensions are small compared to shield thickness) and the optimum material is the most dense material for all geometric shapes (i.e., all values of α). In Fig. 3, the shielded object is comparable to the shield size and we observe a very strong dependence on the geometric shape. The optimum shield material for a cube (i.e., $\alpha = 1$) is the lightest while all other shapes require the most dense material for optimal design. Further enlargement of the shielded object now shows the lightest materials to be optimal, as shown in Fig. 4, where

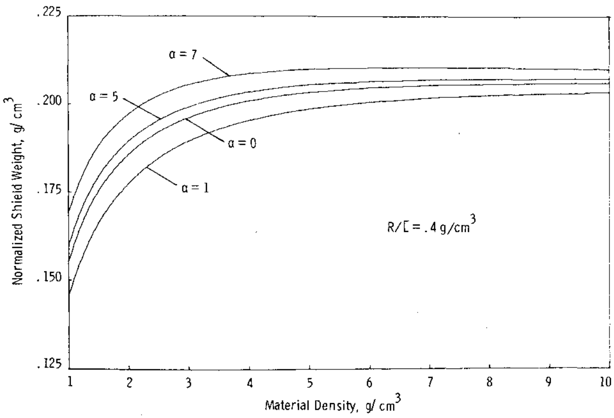

Fig. 4 Normalized shield weight for moderately large shielded regions as a function of shape and material density.

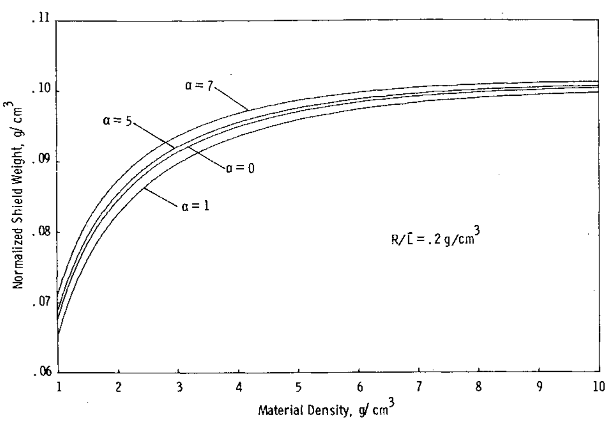

Fig. 5 Normalized shield weight for large shielded regions as a function of shape and material density.

some dependence on shape is still observed. As the shielded object becomes very large, the shield weight vs density curve becomes nearly shape independent over a broad range of shapes, as observed in Fig. 5. The optimal material is always the lightest material for large objects.

Noting the generally strong dependence of optimal material on the shape of the shielded object, we now inquire as to which shapes require the least shield weight. The volume of the shielded region is

$$V_r = \alpha L^3 = 27\ \alpha \bar{L}^3 / (2+\alpha)^3 \tag{12}$$

and the shield weight per displaced weight of the shielded region (i.e., ρV_r) is

$$\frac{W_s}{\rho V_r} = \frac{2}{3}\ \frac{(1+2\alpha)(\alpha+2)}{\alpha}\ r + \frac{4}{9}\ \frac{(2+\alpha)^3}{\alpha}\ r^2 (1 + \tfrac{2}{3} r) \tag{13}$$

where

$$r = F(\rho) R(E) / \rho \bar{L} \tag{14}$$

is the ratio of shield thickness t to average dimension of the shielded region $\bar{L}$. For a given material (i.e., fixed ρ) Eq. (13) gives the amount of shield mass per volume of shielded region for arbitrary geometric shape and fixed ratio of thickness to average dimension. The minimum of Eq. (13) over α then gives the optimum shape. The minimum of Eq. (13) is found to be for $\alpha = 1$ representing a cube. Parameteric curves for Eq. (13) are shown in Fig. 6 for a large range of the ratio of shield thickness to average dimension of the shielded region. Although the cube is the optimum shape, we see that rather

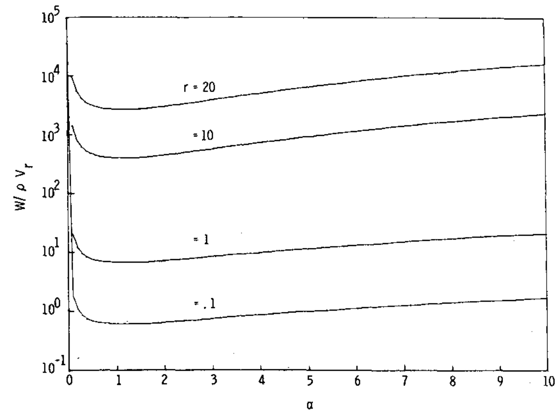

Fig. 6 Shield weight to displaced mass ratio as a function of the ratio of shield thickness to average length of shielded region and shape α. Note that the optimum shape is for $\alpha = 1$ representing a cube in square-box geometry.

large deformations ($0.5 \leq \alpha \leq 2$) from a cube can be made without paying a large weight penalty ($\approx 10\%$).

Discussion

It has been shown for the restricted class of materials in Table 1 that the optimum shield material is generally a strong function of the shape and size of the shielded object. This shape and size dependence can be better understood by considering three geometric limiting cases; namely, a) The shielding of very small objects leads to a shield volume proportional to the shield thickness t cubed, so that

$$W_s \approx 8\ F^3(\rho) R^3(E) / \rho^2$$

which exhibits a strong $1/\rho^2$ dependence and shield weight falls by nearly two orders of magnitude in going from the lightest to the heaviest materials. b) The shielding of very large objects leads to a shield volume proportional to the shield thickness t times the surface area of the shielded object so that

$$W_s \approx F(\rho) R(E) A$$

and the minimum weight corresponds to the smallest value of $F(\rho)$. This dependence is clearly displayed in Fig. 5. c) The shielding of a long, slender object leads to a shield volume proportional to the length of the object times the cross-sectional area of the shield, so that

$$W_s \approx 12 [F^2(\rho)/\rho] R^2(E) \bar{L}$$

and the optimum material is the most dense material. This dependence is approximately that shown in Fig. 2 for $\alpha = 7$.

Although these three limits help us to understand the nature of the shield weight function, the general square-box shield problem exhibits the combined effect of these three limiting cases. The range of behavior of the weight function is characterized by the curves in Figs. 2-5.

Within the manned space program, most shielding problems will involve the shielding of large regions for which shield weight is given by

$$W_s = F(\rho) R(E) A$$

and light materials will be near optimum. In distinction, the most common shielding material used in space applications has been aluminum, which appears in the present analysis as universally suboptimal. In fact, the replacement of aluminum by polyethylene would result in a 35% weight reduction in most space applications. Although a complete replacement of aluminum is not always possible due to structural considerations, a potentially large weight reduction appears possible by proper materials selection.

Conclusions

The optimum shield material is strongly dependent on the geometry of the shielded object. Generally, the optimum material is located at the boundary of the weight curve (i.e., either the lightest or the heaviest materials tend to be optimal). Aluminum, which has been used preferentially for shielding in the space program, appears universally suboptimal. When the shield thickness is small in comparison to the shielded region then the lightest materials, such as polyethylene, water, and polystyrene are optimal. The importance of water as an optimal material is particularly noteworthy for manned missions, since water tanks associated with the life support system could be utilized with little or no weight penalty to protect sensitive on-board materials, such as photographic film. When the shield thickness is large compared to the shielded region then copper or iron are near optimum. Given a volume of goods which must be shielded from the radiation environment, it has been shown that they should be packed as symmetrical about a point as possible to minimize the shield weight. However, modest deviations from exact symmetry do not result in large weight penalties.

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